

# REQUIREMENTS FOR MODEL-BASED POLARIMETRIC DECOMPOSITIONS

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## 1. INTRODUCTION

One of the most complex problems in polarimetric radar imaging is how to interpret the measured co-variance matrix in terms of known scattering mechanisms. Two main approaches are commonly used for this purpose. The first, introduced by Cloude [1] involves calculating the eigenvectors of the measured covariance matrix. This method provides a unique result. However, as shown by van Zyl [2], when these results are interpreted in terms of scattering mechanisms, some approximations have to be made.

A second method, introduced by Freeman and Durden [3], uses a linear combination of covariance matrices from known scattering models to approximate the observed covariance matrix. This method has the advantage that the results are directly interpreted through the component covariance matrices. For this type of decomposition to be accurate, one must require that each component covariance matrix indeed represent a physical scattering process.

## 2. PROPERTIES OF THE COVARIANCE MATRIX

The fundamental measured quantity of a polarimetric radar is the complex 2x2 scattering matrix. The voltage measured by the radar system is proportional to the scalar product of the radar antenna polarization and the incident wave electric field. The power received by the radar is the magnitude of the voltage squared and can be represented in terms of the covariance matrix as described in [1]-[3].

If the covariance matrix corresponds to a single scattering matrix, it is easily shown that it has one positive eigenvalues and the remaining two are identically equal to zero. For multi-look data, the average covariance matrix can have three distinct eigenvalues. However, these eigenvalues must all be non-negative. A simple way to see this is to write the antenna vector in the covariance matrix equation as a linear combination of the eigenvectors of the covariance matrix. Since the covariance matrix is Hermitian, its eigenvectors form an orthonormal set, and we express the antenna vector in this basis. This allows us to show that the received power for any polarization combination is simply a linear combination of the eigenvalues of the covariance matrix, with all the weights positive numbers. Since the power must be non-negative for all antenna combinations, this means the eigenvalues of the covariance matrix must all be non-negative.

## 3. THIS CONTRIBUTION

With this background, we require any decompositions scheme that attempts to represent a measured covariance matrix as a linear combination of covariance matrices to ensure that each of the individual covariance matrices also have positive eigenvalues. This is necessary because each of these matrices should represent a physical scattering process. We show the theory and then show that the Freeman-Durden decomposition does not guarantee physical results, especially in vegetated areas. We then show a suggested procedure for a generic decomposition that ensures physical results. The results are illustrated with examples.

## 4. ACKNOWLEDGMENT

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## 5. REFERENCES

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